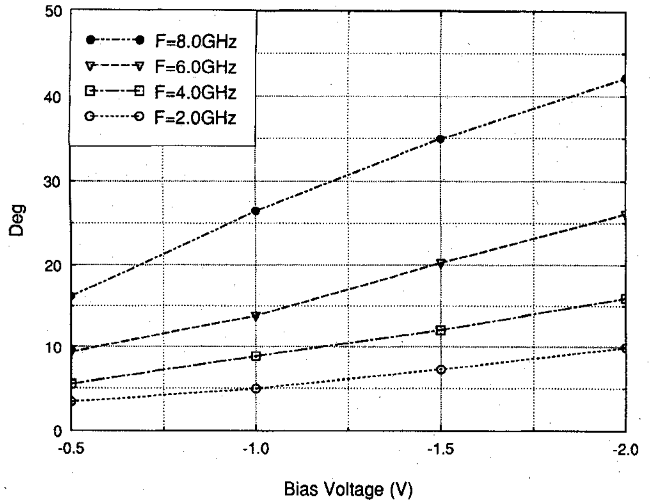

Fig. 5. Phase shift linearity with tuning voltage.

using standard FET structures. The new diode has been characterized and a fit to a standard equivalent circuit has been made. The diode has then been used to realize a very simple loaded line phase shifter with analogue voltage control. The phase shifter has shown excellent characteristics, with very good phase linearity with bias voltage, low insertion loss and low reflection coefficient.

These results have been obtained using a standard foundry process in which the diode active layer has not been optimized for varactor operation. If this was available, lower series resistance and greater capacitance variation could be obtained, leading to even better results with minimal circuit area.

The device could form the basis of other applications, for example in reflection-type phase shifters, where its smaller size and simpler construction could be used to good advantage.

Acknowledgment

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References

[1] Special Issue on Electronic Scanning, *Proc. IEEE*, vol. 56, no. 11, Nov. 1968.

[2] C. Andricos, I. J. Bahl, and E. L. Griffen. "C-Band 6-Bit GaAs monolithic phase shifter," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, no. 12, pp. 1591–1596, Dec. 1985.

[3] I. J. Bahl and P. Bhartia, *Microwave Solid State Circuit Design.* New York: Wiley, 1988.

[4] S. Lucyszyn and I. D. Robertson. "Two-octave bandwidth monolithic analog phase shifter," *IEEE Microwave Guided Wave Lett.*, vol. 2, no. 8, pp. 343–345, Aug. 1992.

[5] E. H. Rhoderick, *Metal-Semiconductor Contacts.* New York: Clarendon Press, 1978.

[6] S. M. Sze, *Physics of Semiconductor Devices.* New York: Wiley, 1981.

[7] N. J. Cronin and V. J. Law. "Planar millimeter-wave diode mixer," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 827–830, 1985.

[8] H. A. Atwater. "Circuit designs of the loaded-line phase shifter," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 626–634, July 1985.

[9] K. Wilson, J. M. C. Nicholas, G. McDermott, and J. W. Burns. "A novel X-Band phase shifter," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1572–8, Dec. 1985.

Permeability Measurement on Composites Made of Oriented Metallic Wires from 0.1 to 18 GHz

Pierre-Marie Jacquart and Olivier Acher

***Abstract*—In this paper, we study the microwave properties of strongly anisotropic materials made of orientated conducting wires. We have developed a broad band method to determine their permeability $\mu_{//}$ parallel to the direction of the wires. We investigate the magnetic properties of strongly anisotropic composites made of different types of paramagnetic and ferromagnetic wires. A simple model is proposed to account for the skin effect, and agrees with our observations. This leads to a unique broad band method for measuring the permeability of thin conducting wires.**

I. Introduction

Numerous studies have been dedicated to the optical and microwave properties of composites materials [1] and [2]. In this paper, we will focus on composites made of parallel conducting wires inserted in an insulating matrix. When the wires are thin enough, the material can be described by an effective medium, and its permittivity and permeability are tensors. The permittivity component for an electric field parallel to the wires is very large, since the wires are conducting, and the permittivity components perpendicular to the direction of orientation of the wires are small, corresponding to an insulating behavior [3]. For that reason, we call these composites strongly anisotropic composites, conducting along one-dimension C1D. Another type of strongly anisotropic composites consists of laminations of alternated insulating and conducting sheets, conducting along two dimensions (C2D) [4]. The experimental determination of the microwave properties of an anisotropic material may be a difficult problem [5] and [6]. In this paper, we present an original method to determine two components of the permeability and permittivity tensors on C1D strongly anisotropic composites. We focus on their magnetic properties and we show that the measurements can be further exploited into determining the intrinsic permeability of the wires in a frequency range where no other measurement procedure exists.

II. Experimental Method

Coaxial line measurements are widely used to determine the microwave properties of isotropic materials [7] and [8]. This broadband method requires standard apparatus and small samples. Since the fundamental mode in the coaxial line—a transverse electromagnetic mode (TEM)—has radial electric field and concentric magnetic field, it is intuitive that a sample consisting of concentric isolated metallic rings behaves as an insulator in the direction of the electric field and allows the penetration of the wave in the composite [Fig. 1(a)]. In fact, we slightly depart from the ring geometry. We wind our thin conducting wires into torus that appear as insulating for the fundamental mode propagating in the coaxial line, and therefore with relatively small permittivity. The reflection and transmission coefficients of the transverse electromagnetic mode on such a sample yield the permittivity $\varepsilon_{\perp}$ of the composite in the direction of the radial electric field, i.e., in the direction perpendicular to the wires,

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P.-M. Jacquart was with the CEA Centre d'Etudes de Bruyères-le-Châtel, Service MA, F-91680 Bruyères-le-Châtel, France. He is now with Dassault Aviation, DGT/DTA/MT, Cedex 300 F-92552 St. Cloud Cedex.

O. Acher is with the CEA Centre d'Etudes de Bruyères-le-Châtel, Service MA, F-91680 Bruyères-le-Châtel, France.

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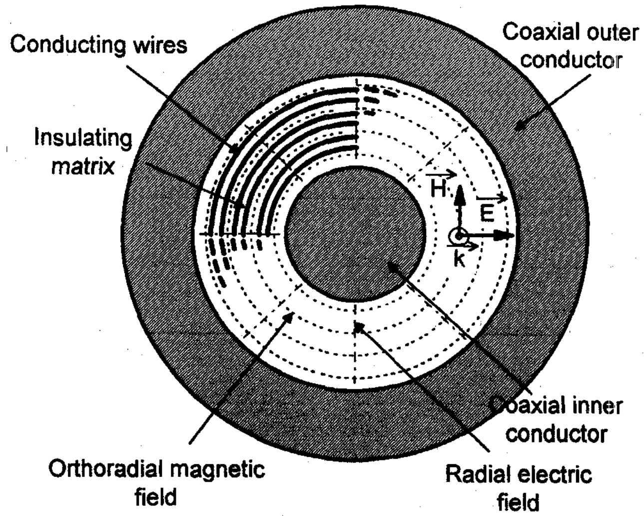
and its complex permeability $\mu_{//}$ parallel to the wires. In this paper, we particularly focus on the experimental measurement of $\mu_{//}$. Details on the permittivity measurements are reported elsewhere [3]. The wires, supplied by Isabellenhütte [9], are recovered by a thin layer of insulating varnish. The conducting wires are essentially $Ni_{80}Cr_{20}$, copper, nickel, and $Ni_{70}Fe_{30}$ ones. The $Ni_{80}Cr_{20}$ alloy is not ferromagnetic, and $Ni_{70}Fe_{30}$ is ferromagnetic. The radius of the wires is noted a and ranges from 20–100 μm. The electrical resistivity ρ given by the manufacturer is between 2–110 $\mu\Omega$.cm. The composite samples are made by winding and gluing the isolated wires into torus, and then are machined to the precise dimensions of an APC-7 coaxial line (7 mm outer diameter, 3 mm inner diameter) [Fig. 1(b)]. Torus thickness ranges from 0.7–3 mm. The volume fraction q of conductor in the torus is deduced from density measurements with an uncertainty lower than 7%. The microwave properties of the torus are measured using a conventional Hewlett-Packard APC-7 system developed for isotropic materials. It yields values of $\mu_{//}$ with a precision better than 8% for frequencies higher than 500 MHz. For frequencies in the 100–500 MHz range, the precision is not better than 20% due to the very small thickness of the sample compared to the wavelength. The upper measurement frequency is fixed by the capabilities of the network analyzer used, and also by the propagation of higher order modes in the coaxial line. It would be possible to extend the upper frequency bound by using a proper network analyzer and a coaxial line with smaller diameter.

III. Experimental Results

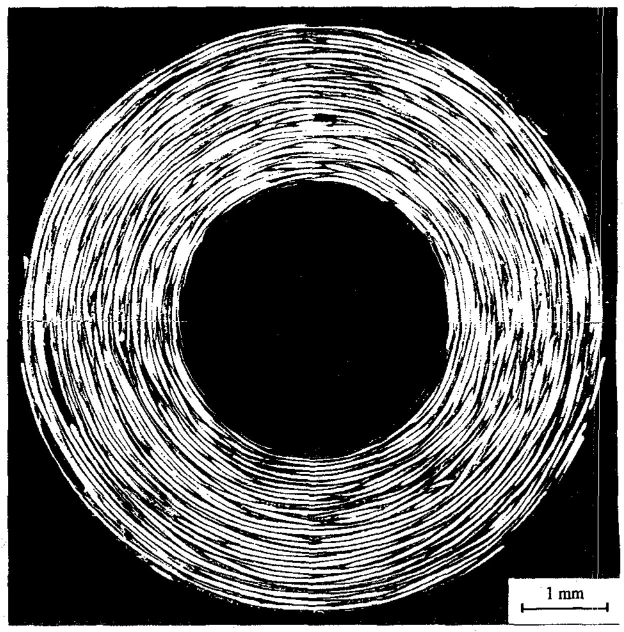
A first concern was to validate our measurement method. We compared experimental results obtained with our method to rectangular waveguide measurements, as reported elsewhere [3] and [10]. The agreement between the two methods is satisfactory, especially for the permeability. However, waveguide measurements on strongly anisotropic C1D materials suffer limitations due to the propagation of parasitic modes [10]. Besides, these comparisons could be performed on a limited frequency range, due to the limited band of the waveguides.

Fig. 2 shows the variation of the complex permeability $\mu_{//} = \mu'_{//} - j\cdot\mu''_{//}$, of torus samples made of conducting wires wound in a dielectric matrix, with frequency. The composites were manufactured with copper wires with 50 μm diameter, resistivity $\rho = 1.72\,\mu\Omega$.cm (curve a), with $Ni_{80}Cr_{20}$ wires, with 20 μm (curve b) and 50 μm (curve c) diameter and $\rho = 108\,\mu\Omega$.cm, and with Ni wires (curve d) with 50 μm diameter and $\rho = 9\,\mu\Omega$.cm. Their volume fraction q of conductor ranged from 37–42%. Fig. 2 indicates the existence of magnetic losses ($\mu''_{//} \neq 0$) even in samples composed of nonmagnetic inclusions embedded in a dielectric matrix. The apparition of such magnetic losses can be related to eddy currents in the inclusions [11]. On this figure, three trends of $\mu_{//}(f)$ are clearly displayed.

1) $\mu'_{//}(f)$ and $\mu''_{//}(f)$ are affected by the magnetic properties of the wires. The modulus of $\mu_{//}$ is significantly higher than unity at low frequency for the ferromagnetic-based sample (curve d), and the magnetic losses are higher than on the samples made of nonmagnetic wires, in the 0.1–2 GHz range.
2) For torus obtained by winding highly resistive and nonmagnetic wires ($\rho \geq 100\,\mu\Omega$.cm), the permeability $\mu''_{//}$ presents a maximum at a frequency which increases as the radius of the wires decreases (curves b and c). On the other hand, for low resistivity wires (a few $\mu\Omega$.cm) like copper ones, this frequency seems to be less than 0.1 GHz (curve a) and $\mu''_{//}$ tends to zero in the 0.1–18 GHz range.
3) For frequencies higher than the frequency corresponding to the maximum of $\mu''_{//}$, and for a given wire size, the real part of permeability $\mu'_{//}$ tends to an asymptotic value depending on the wire volume fraction q that is close to $(1 - q)$ (curves b and c) whereas $\mu''_{//}$ tends to zero.



(a)



(b)

Fig. 1. (a) Sketch of a strongly anisotropic composite made of oriented conducting wires adapted to coaxial line geometry. The propagation vector **k**, and **E** and **H** fields corresponding to the fundamental mode in the coaxial line are also sketched.(b) Micrograph of the surface of a strongly anisotropic composite adapted to APC-7 coaxial measurement.

Note that $\mu'_{//}(f)$ and $\mu''_{//}(f)$ exhibit some perturbations for frequencies lower than 0.5 GHz due to inaccuracies on the measurements, as we discussed above.

So, Fig. 2 indicates that the magnetic properties of strongly anisotropic composites made of orientated conducting wires are related to the characteristics a, ρ, and μ_i of the particles.

IV. Theoretical Predictions

One of the most successful methods of treating the properties of inhomogeneous materials has been an effective-medium or self-consistent approach. In the static limit, the mean field solution was

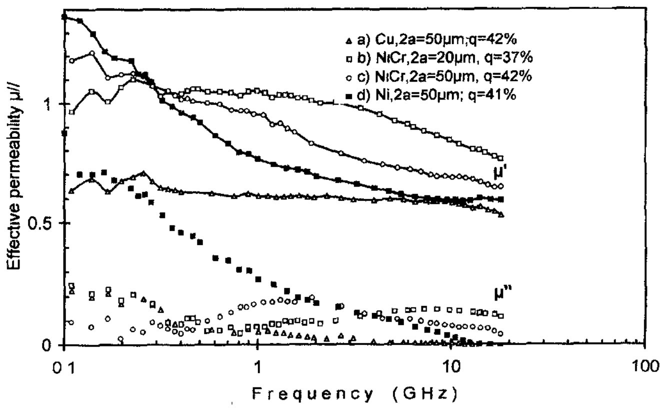


Fig. 2. Microwave permeability $\mu//$ of strongly anisotropic C1D composites made of oriented paramagnetic wires (a, b, c) or ferromagnetic wires (d).

given by Maxwell–Garnett and Bruggeman [12] and [13]. Nowadays, these effective medium theories (EMT) are widely used to treat the magnetic or electric properties of randomly inhomogeneous materials [1] and [14]. They give expressions for the permeability of the composite that are independent of the particle size and conductivity. These theories require that the inclusions are small compared to the wavelength in the inclusions.

The dynamic permeability $\mu_{//}$ of such strongly anisotropic materials measured in the 0.1–18 GHz frequency range exhibits a strong dependence upon the wire radius and electrical resistivity. This indicates that EMT's can not describe the microwave properties of these materials. This dependence is due to the additional magnetic moment induced by eddy currents in the conducting wires and it is therefore necessary to include these effects in an extension of the aforementioned theories. In our composites, the diameter of the inclusions is small compared to the free space wavelength, but it may be large compared to the wavelength in the inclusions.

A. The Quasi-Static Case

To extend an EMT to the quasi-stationary case, i.e., when eddy currents in the metallic inclusions are important, we need to be a little more specific. We consider a composite made of conducting cylinders (electrical resistivity ρ, radius a, and concentration q) which are embedded in an insulating and nonmagnetic matrix (permeability $\mu_m = 1 - j.0$). The inclusions are orientated in the same direction (directional ordering), and they are electrically isolated, but with random position in the matrix. They are supposed to have an infinite length in comparison with their radius a. Their microwave magnetic behavior is described by a scalar permeability μ_i. The inclusions are parallel to the external harmonic magnetic field $\vec{H}_0$ with pulsation ω, as it is the case for a wound torus in a coaxial line with TEM mode propagating. $\vec{H}_0$ is approximately uniform around the wire, since the diameter of the wire is very small compared to the wavelength in the composite. As the tangential components of the electric and magnetic fields are continuous across metal/host interface, the field $\vec{H}_i$ inside a cylinder at a distance r of its axis, is related to the external one by [11]

$$\vec{H}_i(r) = \frac{J_0(k_i.r)}{J_0(k_i.a)}.\vec{H}_0 \tag{1}$$

where k_i corresponds to the wavevector in the conducting material

$$\begin{aligned} k_i &= k_i' - j.k_i'' \\ &= \sqrt{\frac{-j.\omega.\mu_0.\mu_i}{\rho}} \end{aligned} \tag{2}$$

with $\exp(+j\omega t)$ time convention. $J_0(x)$ is the Bessel function of order zero.

The magnetic field inside the wire is parallel to the cylinder axis, so the electrical field and the eddy currents are azimutal. If we define the artificial, or apparent, permeability μ_a of a conducting cylinder, submitted to an external field, as the spatial average of the magnetic induction divided by the applied field $\vec{H}_0$, we obtain [3] and [15]

$$\begin{aligned} \mu_a &= A.\mu_i \\ &= \frac{2}{k_i.a}.\frac{J_1(k_i.a)}{J_0(k_i.a)}.\mu_i \end{aligned} \tag{3}$$

with $J_1(x)$ the Bessel function of order one. We call A the function of attenuation of the conducting inclusion subject to an applied magnetic field. In fact, the expression of A depends on the geometry of the inclusion and on the direction of orientation of the external field in comparison with the inclusion axis [11] and [16]. It is of interest to note that if $\mu_i' = 0$ then $\mu_a' = 0$; this indicates that the frequency for which μ_i' changes sign should not be affected by skin effect.

As proposed for spherical inclusions [2] and [17] one extends EMT's by replacing the static polarizability of a cylinder, with permeability μ_i, by its quasi-static polarizability, with permeability μ_a, subject to a time varying magnetic field. For wires parallel to the applied field $\vec{H}_0$, their demagnetizing coefficient is equal to zero; Maxwell–Garnett and Bruggeman EMT's now yield

$$\mu_{//} = q.A.\mu_i + (1-q).\mu_m. \tag{4}$$

Therefore, the effective permeability $\mu_{//}$ of anisotropic materials made of oriented conducting cylinders embedded in a dielectric matrix depends on their size and electrical resistivity.

B. Modeling of Data

The permeability μ_i of conducting inclusions is well known for paramagnetic ones and is equal to $1 - j.0$. In contrast, μ_i is not well defined for ferromagnetic inclusions. In this case, μ_i is essentially governed by the domain wall motion and the gyromagnetic response of the ferromagnetic wires [15]. These contributions can be drastically affected by the internal stress via magnetostrictive effects, or by the dimensions of the wires by demagnetizing effects. As a consequence, C1D composites manufactured from nonmagnetic wires ($\mu_i = 1 - j.0$) are more suitable to compare our permeability measurements to theoretical predictions.

We present in Fig. 3 the evolution of the effective permeability $\mu_{//}(f) = \mu_{//}'(f) - j.\mu_{//}''(f)$ computed from (4), for two materials presented on Fig. 2, in the 0.1–18 GHz range. This corresponds to materials made with wires of copper ($\rho = 1.72\,\mu\Omega$.cm, $a = 25\,\mu$m, and q = 42%) (curve c) and $Ni_{80}Cr_{20}$ ($\rho = 108\,\mu\Omega$.cm, $a = 10\,\mu$m, and q = 37%) (curve d). Fig. 3 shows a very good agreement between theoretical predictions and experimental results. This is a strong argument in support of our measurement procedure and of our finite frequency extension of the EMT. It addresses the objection that our coaxial measurements are performed on spiral-wound samples, and not on strictly coaxial geometry. However, one should keep in mind that the equivalence of properties between wound and perfectly concentric torus has a limited range of validity. It is expected no longer to be valid at low frequency, where the wound torus will act as a short if the conducting wire is continuous between the inner and outer conductors of the coaxial line.

V. Permeability of μ_i Ferromagnetic Wires

We can use the theoretical approach presented before to determine the permeability $\mu_i(f)$ of ferromagnetic wires from experimental determinations of the permeability $\mu_{//}$ on a wound torus. Ferromagnetic

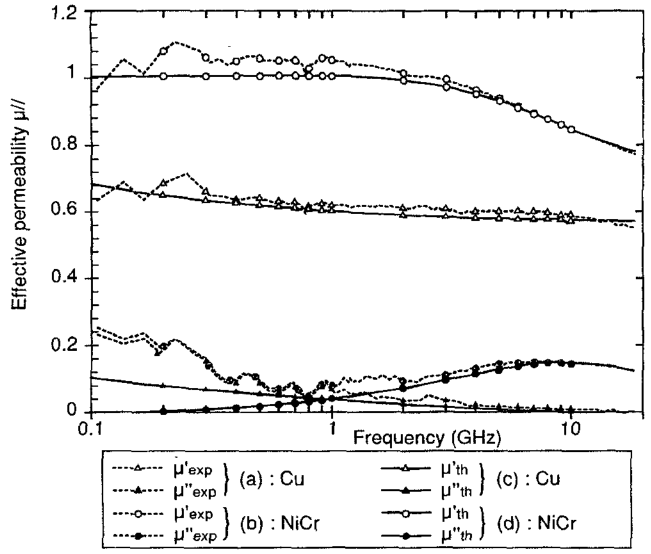


Fig. 3. Effective permeability $\mu//$ of strongly anisotropic C1D composites made of Cu and NiCr wires. (a) and (b) Measured on torodal samples with a coaxial line technique (same as Fig. 2). (c) and (d) Calculated using an extension of an effective medium theory.

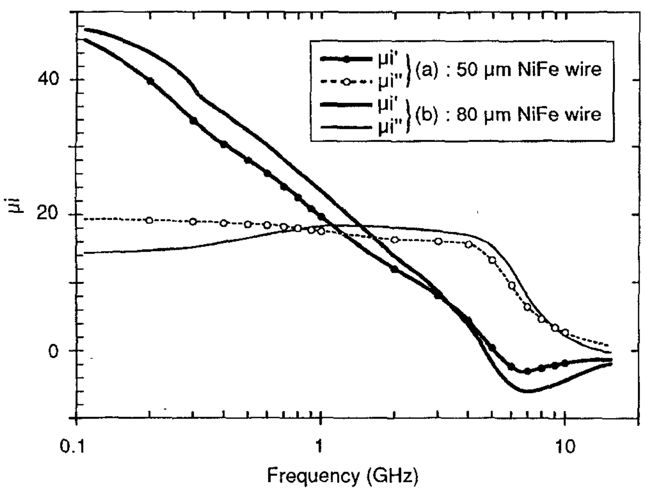


Fig. 4. Complex permeability $\mu\imath$ of ferromagnetic Ni70Fe30 wires ($\rho = 33\mu\Omega$.cm) with (a) 50μm diameter and (b) 80μm diameter, determined from microwave measurements performed on C1D wound torus.

$Ni_{70}Fe_{30}$ wires of different diameters were wound into C1D torus suitable for coaxial line measurements. The permeability $\mu_\imath$ of the $Ni_{70}Fe_{30}$ wires was computed from the experimental measurements of $\mu_{//}$ using (4). Fig. 4 shows that the permeabilities $\mu_\imath$ of the 50 and 80 μm inclusions are nearly the same. The gyromagnetic resonance frequency [defined by $\mu'(f) = 1$] is about 4.5 GHz. The ability of our technique to study gyromagnetic resonance of thin ferromagnetic wires, in the absence of an external DC field, is particularly valuable in comparison with conventional ferromagnetic resonance experiments. In particular, it is possible to study the influence of the magnetic domain structure on the gyromagnetic response of wires, and it offers unique high frequency and broad band ability. The samples are relatively small compared to samples required for rectangular waveguide measurements.

It is clear that this method can be applied to other types of anisotropic materials [4], provided they can be bent and manufactured into a composite with nearly cylindrical symmetry. When homogenization laws offer a good description of the permeability of the composite, this method allows the determination of the permeability of the inclusion along one direction, as it has been shown already for thin films [18].

VI. Conclusion

We investigated the microwave properties of strongly anisotropic composites conducting along C1D, made of orientated metallic wires embedded in an insulating matrix. We presented a new experimental method to investigate their microwave properties, based on standard coaxial-line measurement apparatus. The permeability $\mu_{//}$ of C1D composites has been investigated in further details. A model without any adjustable parameter has been proposed to account for the dependence of $\mu_{//}$ with the resistivity, the radius, the permeability, and the volume fraction of the wires. We report experimental results that show a very good agreement with this model. This leads to a new broad-band method to measure the permeability of wires with a good precision at frequencies far higher than those accessible with already existing techniques.

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References

[1] Y. Yagil, P. Gadenne, C. Julien, and G. Deutscher, "Optical properties of thin semicontinuous gold films over a wavelength range of 2.5–500 μm," *Phys. Rev. B*, vol. 46, p. 2503, 1992, and references therein.

[2] D. Rousselle, A. Berthault, O. Acher, J. P. Bouchaud, and P. G. Zérah, "Effective medium at finite frequency: Theory and experiment," *J. Appl. Phys.*, vol. 74, p. 475, 1993, and references therein.

[3] P. M. Jacquart, "Etude et caractérisation hyperfréquence des propriétés électromagnétiques de matériaux inhomogènes très anisotropes: Mélanges conducteurs-diélectriques," *These de l'Université Pierre et Marie Curie—Paris*, 1994.

[4] O. Acher, P. M. Jacquart, J. M. Fontaine, P. Baclet, and G. Perrin, "High impedance anisotropic composites manufactured from ferromagnetic thin films for microwave applications," *IEEE Trans. Mag.*, vol. 30, p. 4533, 1994.

[5] G. Mazé, "Thèse de l'institut national Polytechnique de Grenoble," 1991.

[6] F. Gardiol, "Anisotropic slabs in rectangular waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 18, pp. 461–467, 1970.

[7] A. M. Nicolson and G. F. Ross, "Measurement of the intrinsic properties of materials by time domain techniques," *IEEE Trans. Instrum. Meas.*, vol. 17, p. 395, 1968.

[8] W. W. Weir, "Automatic measurement of complex dielectric constant and permeability at microwave frequencies," *Proc. IEEE*, vol. 62, p. 33, 1974.

[9] Isabellenhütte, technical documentation.

[10] P. M. Jacquart, O. Acher, and S. Lefrançois, "Etude de composites très anisotropes à base de fils conducteurs orientés," in *Proc. 3rd JCMM*, Brest, 1994.

[11] L. D. Landau and E. M. Lifchitz, *Electrodynamics of Continuous Media*. New York: Pergamon, 1986.

[12] J. C. Maxwell Garnett, *Philos. Trans. R. Soc. Lond.*, vol. 203, p. 385, 1904.

[13] D. A. G. Bruggeman, *Ann. Phys. Leipz.*, vol. 24, p. 636, 1935.

[14] W. M. Robertson, G. Arjavalingam, and S. L. Shinde, "Microwave dielectric measurements of zirconia-alumina ceramic composites: A test of the Clausius–Mossotti mixture equations," *J. Appl. Phys.*, vol. 70, p. 7648, 1991.

[15] O. Acher, P. M. Jacquart, and C. Boscher, "Investigation of high permeability of thin amorphous wires," *IEEE Trans. Mag.*, vol. 30, p. 4542, 1994.

[16] S. M. Rytov, "Electromagnetic properties of a finely stratified medium," *Sov. Phys. JETP*, vol. 2, p. 466, 1956.
[17] G. D. Mahan, "Long-wavelength absorption of cermets," *Phys. Rev. B*, vol. 38, p. 9500, 1988.
[18] O. Acher, J. L. Vermeulen, P. M. Jacquart, J. M. Fontaine, and P. Baclet, "Permeability measurement on ferromagnetic thin films from 50 MHz up to 18 GHz," *J. Magn. Mat.*, vol. 136, p. 269, 1994.

Dispersion Characteristics of Cylindrical Coplanar Waveguides

Hsin-Cheng Su and Kin-Lu Wong

Abstract— **A full-wave analysis of the coplanar waveguide (CPW) printed on a cylindrical substrate is presented, and the dispersion characteristics of the cylindrical CPW are studied. Numerical results of the effective relative permittivity are calculated using a Galerkin's moment-method calculation. Experiment is also conducted, and the measured data are in good agreement with the theory.**

I. Introduction

Although coplanar waveguides (CPW's) provide some advantages over microstrip lines [1], such as easier connection with integrated active devices and no via holes for grounding, etc., it is noted that the studies of conformal CPW's receive much less attention than conformal microstrip lines. The microstrip lines mounted on curved surfaces, such as cylindrical [2], [3], or elliptical [4] bodies, have been extensively studied. However, to the best of our knowledge, the studies of conformal CPW's have not yet been reported in the open literature. To increase the application of CPW's on curved surfaces and analyze the dispersion characteristics of conformal CPW's, we present in this paper a full-wave analysis of the CPW printed on a cylindrical substrate. Numerical results of the frequency-dependent effective relative permittivity are calculated and analyzed. Measured data are also presented for comparison with the calculated results, and the curvature effect on the effective relative permittivity of the CPW is discussed.

II. Theoretical Formulation

Fig. 1 shows the geometry of a cylindrical coplanar waveguide. The CPW is assumed to be infinitely long, and the radius of the cylindrical ground plane is b. The thickness and relative permittivity of the substrate is $h(= b - a)$ and ε_2, respectively. The width of the signal strip is S, and the ground-to-ground spacing is d. The region $\rho < a$ is assumed to have a relative permittivity ε_1. Outside the ground plane is air with free-space permittivity ε_0 and permeability μ_0.

To begin with, the spectral-domain Helmholtz's equations in each region of the structure are solved, which gives the expressions of the spectral amplitudes of the electric and magnetic fields in inner region ($\rho < a$), substrate layer ($a \le \rho \le b$), and outer region

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The authors are with the Department of Electrical Engineering, National Sun Yat-Sen University, Kaohsiung, Taiwan.

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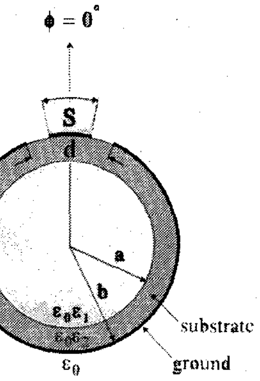

Fig. 1. The geometry of a cylindrical coplanar waveguide.

($\rho > b$). Then, by applying the equivalence principle [5], the slot region between the signal strip and the ground can be closed off and replaced by an equivalent magnetic surface current density $\vec{M}_s(= M_\phi\hat{\phi} + M_z\hat{z})$ at (b^-, ϕ, z) and $-\vec{M}_s$ at (b^+, ϕ, z). When imposing the boundary conditions of the structure and manipulating the derived field components, we can relate the difference of the tangential magnetic fields at $\rho = b^-$ and $\rho = b^+$ to the magnetic surface current density as

$$\begin{bmatrix}\Delta\tilde{H}_\phi \\ \Delta\tilde{H}_z\end{bmatrix} = [\overline{\overline{G}}^{(s)} - \overline{\overline{G}}^{(a)}] \cdot \begin{bmatrix}\tilde{M}_\phi \\ \tilde{M}_z\end{bmatrix} = \begin{bmatrix}0 \\ 0\end{bmatrix} \qquad (1)$$

where $\overline{\overline{G}}^{(s,a)} = \hat{\phi}\tilde{G}^{HM(s,a)}_{\phi\phi}\hat{\phi} + \hat{\phi}\tilde{G}^{HM(s,a)}_{\phi z}\hat{z} + \hat{z}\tilde{G}^{HM(s,a)}_{z\phi}\hat{\phi} + \hat{z}\tilde{G}^{HM(s,a)}_{zz}\hat{z}$ is dyadic Green's functions showing the H_ϕ or H_z fields on the substrate (s) or air (a) sides of the slot region due to a unit M_ϕ or M_z at the slot region. The expressions of the Green's functions are derived in the Appendix, and the tilde denotes a Fourier transform. ΔH shows the difference of the tangential magnetic fields at $\rho = b^-$ and at $\rho = b^+$, which must be zero to satisfy the boundary condition that the continuity of the tangential magnetic fields at the slot region must hold.

And, due to the assumption that the cylindrical CPW is infinitely long, the magnetic surface current at the slot region can be assumed to have a traveling-wave form of $e^{j\beta z}$, where β is the effective propagation constant to be determined. In this case we have

$$\vec{M}_s = e^{j\beta z}[\hat{z}M_z(\phi) + \hat{\phi}M_\phi(\phi)]. \qquad (2)$$

To apply the moment method, we choose N rooftop basis functions of the form

$$M_{\phi n} = 1 - \frac{2|\phi - \phi_{\phi n}|}{D_t}, \quad |\phi - \phi_{\phi n}| < \frac{D_t}{2} \qquad (3)$$

with

$$\phi_{\phi n} = \frac{S}{2b} + \frac{nD_t}{2},$$
$$D_t = \frac{d - S}{b(N+1)}$$